SCALAR PARTICLES CREATION RATE IN AN EXPANDING UNIVERSE

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Quantum field theory in curved space-time, where the gravitational field is treated as a classical solution of Einstein field equations and matter and radiation fields are quantized in that background, predicts the creation of particles in non-static situations, such as in an expanding universe (see refs. 1, 2 y 3 for recent reviews).

One of the crucial problems of this theory is that of defining the particle concept, which in the flat space case is given by the plane wave descomposition into positive and negative frequency parts of the field. This trouble can be avoided defining the particle number only in asymptotically static cases ("in-out" theories), but this procedure does not allow to reintroduce the created matter as a source in Einstein equations, in order to study its effects on the evolution of the universe. Thus it is generally preferred to work with quantities such as the energy-momentum tensor, defined directly from the field independently of the existence of a basis with well defined particle number for the quantum states. However, these quantities are generally divergent, and must be renormalized with ad-hoc methods (4).

In this work, on the contrary, we make use of the particle model for curved space-time recently proposed by one of the authors (M. Castagnino) and R. Weder (5), called the "quantum equivalence principle" in order to develop the general formalism to find the number of created particles as a function of time in an expanding universe.

The quantum equivalence principle allows an invariant descomposition of a field into its positive and negative frequency parts assuming that the Green functions for the Klein-Gordon equation in curved space coincide with the respective flat space-time Green function in a normal coordinate system (i.e. in the freely falling system that most closely resembles to cartesian coordinates). At ref. (6), by means of this principle, the boundary conditions over each Cauchy surface for the positive and negative frequency solutions are obtained up to second order in an expansion in powers of the hubble coefficient $H = \dot{a}/a$ for a Robertson-Walker spatially flat metric

$$ds^2 = g_{ij}\,dx^i dx^j = dt^2 - \sum_{\alpha=1}^{3} a^2(t)\,dx^\alpha dx^\alpha \qquad (1)$$

We consider a scalar field $\phi(x)$ satisfying the Klein-Gordon equation generalised to curved space

$$\left(\Delta - m^2 - \xi R\right)\phi(x) = 0 \qquad (2)$$

Where $\Delta = -g_{ij}\nabla_i\partial_j$ is the Laplace operator, with g_{ij} the metric tensor, ∇_i the covariant derivative and $R = 6(\dot{H}+2H^2)$ the scalar curvature. ξ is an arbitrary parameter that must be taken equal to 1/6 in order to make the equation conformally invariant in the massless case. We find the appropiate linear combination of orthonormalized solution of eq. (2) that satisfies the quantum equivalence principle over each Cauchy surface Σ_τ: $\{t = \tau = \text{const.}\}$. Thus we obtain the appropiate basis $\{\psi_{\underline{k}}(\mathbf{x}),\ \psi^*_{\underline{k}}(\mathbf{x})\}$ such as to interpret the coefficients of the field descomposition

$$\phi(x) = \int d^3\underline{k}\,\left[a_{\underline{k}}\,\psi_{\underline{k}}(x) + a^\dagger_{-\underline{k}}\,\psi^*_{-\underline{k}}(x)\right]$$

as creation and annihilation operators. We then find the Bogoliubov transformation relating the positive frequency solutions at Σ_τ with positive and negative frequency solutions at another surface Σ_{τ_0}

$$\psi^{(\tau)}_{\underline{k}}(\underline{x},t) = \alpha_k(\tau_0,\tau)\,\psi^{(\tau_0)}_{\underline{k}}(\underline{x},t) + \beta_k(\tau_0,\tau)\,\psi^{(\tau_0)*}_{-\underline{k}}(\underline{x},t)$$

and then the number of created particles between τ_0 and τ

$$N_{\underline{k}}(\tau) = N_{\underline{k}}(\tau_0) + |\beta_{\underline{k}}|^2\left[1 + 2N_{\underline{k}}(\tau_0)\right] \qquad (3)$$

$$|\beta_k(\tau_0,\tau)|^2 = L^2(\tau) + L^2(\tau_0) - 2L(\tau)L(\tau_0)\cos\left[2\int_{\tau_0}^{\tau}\omega_{\underline{k}}\,dt\right] \qquad (4)$$

with

$$L(k,t) = \frac{1}{4\omega^2_{\underline{k}}}\left(\frac{m^2}{2}H^2 + \frac{R}{6}\left(\frac{m^2}{2} + \omega^2_{\underline{k}}\right)\right)$$

$\omega_{\underline{k}} = \left[\frac{k^2}{a^2} + m^2\right]^{1/2}$ is the energy, $\underline{k}/a$ the linear momentum and m the mass of the particle. This result is valid when the condition $H/\omega_{\underline{k}}$ is satisfied, because otherwise the expansions made to obtain the boundary conditions would be meanningless.

The present creation rate is negligeable compared to the already existent matter density. Indeed we observe that for most common cosmological models $a(t) = At^\varepsilon$ with $\varepsilon<1$, then if $\tau_0 \ll \tau$, we have $L(\tau_0) \gg L(\tau)$. Thus the number of created particles between two very widely separated instants ocurrs mainly in the neigbourhood of the initial one, and does not depend on the particular evolution throughout the time range. We then evaluate the flux of created particles in terms of R_0 and a_0/a, the scalar curvature at τ_0 and the ratio of the radius of the universe at τ_0 and at the present time. Considering $\tau_0 \ll \tau$ and having into account that for the interesting cases $m \ll k/a_0$ (the particles were ultrarelativistic at the creation instant) we obtain

$$\Phi(T)\,dT = \frac{\hbar}{16\,(2\pi)^3 c^2}\left(\frac{R_0}{6}\right)^2\cdot\left(\frac{a_0}{a}\right)^4\cdot\frac{dT}{T^2+m\hbar T} \qquad (5)$$

giving the present flux per kinetic energy (T) interval, per unit solid angle.

Experimental evidence for diffuse cosmic X and γ ray background (7) reveals a spectrum given by $\Phi(\varepsilon) = 25\, E^{-2}\ \frac{\text{kev}}{\text{cm}^2\text{s}}$, the same dependence on the energy as our formula (5) in the massless case. Assuming the standard hot big-bang model for the evolution of the universe, taking the age of the universe as Hubble time $t_H = H^{-1} \sim 10^{18}$s., we put $\frac{a_0}{a} = \frac{\tau_0^{1/2}}{t_H^{2/3}}\cdot\frac{t_D^{2/3}}{t_D^{1/2}}$ where t_D denotes the time when matter and radiation became decoupled, approximately 10^{13} s. Assuming $R_0 \sim \frac{1}{\tau_0^2}$, without specifying strictly the time dependence of the radius of the universe in the neigbourhood of τ_0, and equating (5) to the experimental data (although our model is for scalar particles) we obtain $\tau_0 \sim 3 \times 10^{-43}$s., which is of the order of Planck time,

$$t_P = \left(\frac{G\hbar}{c^5}\right)^{1/2} \sim 10^{-43}\,\text{s}.$$

Heuristic arguments show (8) that this semiclassical theory can be applied just until Planck time without being necessary to quantify gravitation. The condition $M/\omega_k < 1$ is also verified in the situation considered. Nevertheless the result is not acceptable, because if the particles would have been created before the decouple of radiation with matter the created photons would became thermalized, and they would be present today at the 3K radiation background.

Although this negative result we expect that further improvements of the model, and its aplications to higher spins, would eventually allow to formulate a more realistic cosmological model.

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